

# Early Year's Geometry

## — Children's Conception According 'Transformation'

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**Abstract** - We have examined drawings of figures drawn by children from 3 to 5 years old. We pursue the problem of the quality of learning by way of analyzing children's conception of space. We had shown them figures (using two cups) as a model. One cup was placed in ordinal way, the other in reverse. How children drew figures concerning domains. Why differences occur when children drew one and the same model? Our research to the problem of cognition of 'Transformation' by children is carried out by the standpoint of projective geometry.

### INTRODUCTION

Many teachers pay attention only to the result of teaching and educational effects, but have a few interests to the method of thinking by children. They ignore the progress and process of investigation, or the stumbling in course of their thinking before arriving at a result.

When children are confronted with difficulties, first and foremost they rush in their shelters. These shelters are their abilities to do their best. They mobilize all their abilities, they think out their own decisions. But it is difficult for children how to express the contents in their mind. Therefore we should support them by the appeal to their process of thinking. Development of learning begins from noticing their own troubles of understanding by themselves.

Our attempt by the method of drawing-analysis may offer some hints to find out the secret of their process of thinking.

### TASK AND PROCEDURE

Children from 3 to 5 year old are told to draw the model (figure 1), using two cups as a model. They were set in different ways. One was in an ordinary way, the other was upside down. The color of each domain of cups was pink, yellow, red and blue. When we looked at the left cup, we couldn't see the domain of the inside bottom.

**Table 1: Sam total of children and sheets**

2009.2,24 – 3,18 In Sendai, KATA nursery school numbers plane model cubic model

3 years old	20	27	52
4 years old	24	23	45
5 years old	23	34	60

### Method of this experiment

\* Pretest - We asked children to draw the plane model first, and then cubic model without any suggestion.

\* Game - After the drawing, they played the following game. The game was called pole asunder game.

\* Posttest – After playing the game, they were asked to draw the same cubic models again.

### MODEL FOR DRAWING

How children draw the model? We analyze this problem applying the method of projective geometry.

**Figure 1: the plane model**

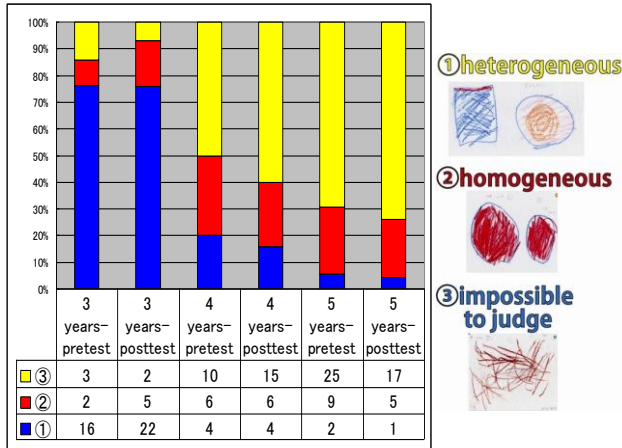


**cubic model**



### 1 Characteristics of drawings of children 1.1 Distinction of the qualities in the drawing

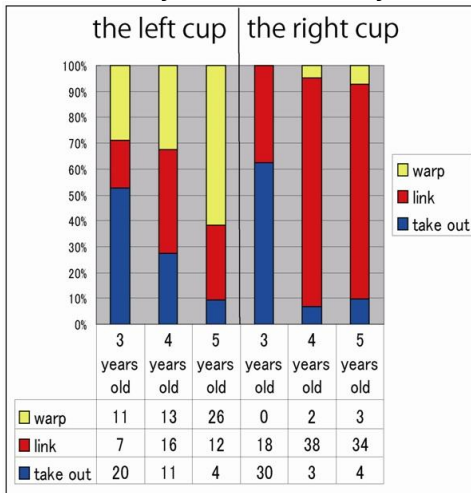
**Table 2: distinction of the qualities of two cups**



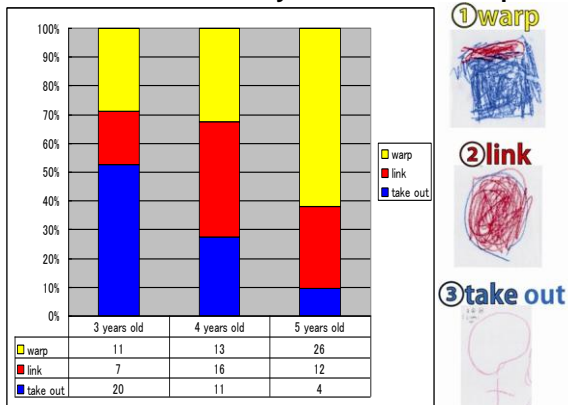
For the drawing of cups from different two standpoints, children felt many difficulties. By a child, we could find differences of understanding between the two models. But the quality of understanding was getting raised gradually by age. In the case of the drawing of the posttest by 4 and 5 years children, a child showed the understanding of each model heterogeneously. In case of the drawings by 3 years, children drew two cups as one cup without distinguishing them, or drew them in two circles.

**1.2 How children drew differently the model of two cups?**

**Table 3: The way of draw differently two cups**



**Table 3-a: The way of draw the left cup**



- 1) take out – simple picking out of domains
- 2) link – connection of two or many domains
- 3) warp – transformation from circle to ellipse

**Figure2: take out (1)**



**Figure3: take out (2)**



**Figure4: take out (3)**



**Figure 5: link (1)**



**Figure 6: link (2)**



**Figure 7: link (3)**



**Figure8: warp (1)**



**Figure9: warp (2)**



**Figure10: warp (3)**



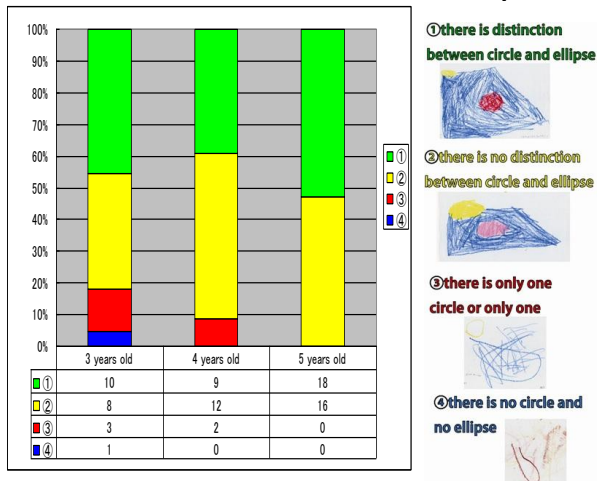
[Result]

Left cup: Transformation of a domain is changing one after another: 'take out'—'link'—'warp' by age (Table 3).

Right cup: Connection of three domains is derived differently by age.

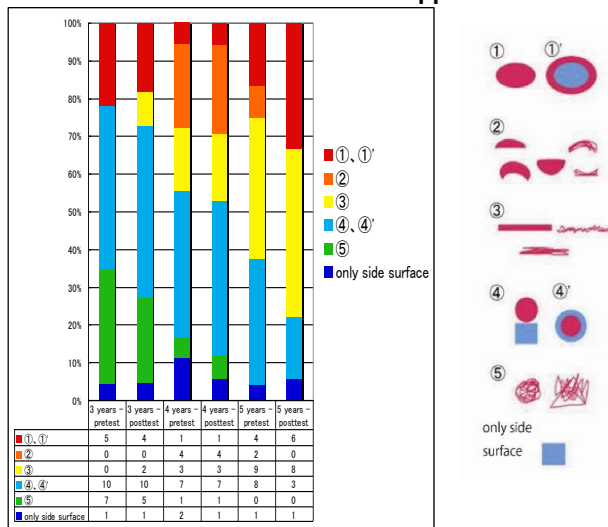
### 1.3 How children drew the plain model in circle and ellipse?

Table 4: distinction between circle and ellipse



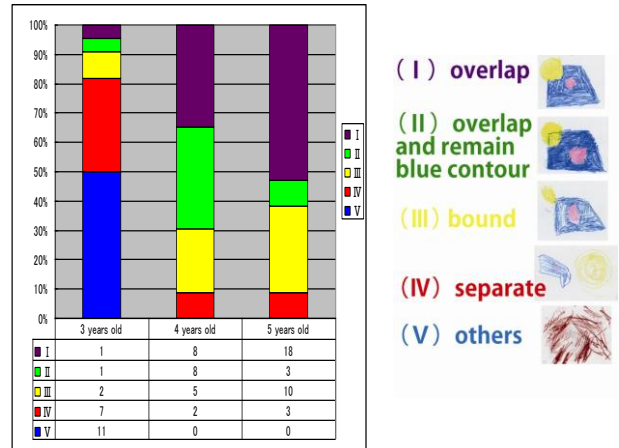
- ① By plain model, over 40 % children could draw the model correctly distinguishing circle and ellipse. But they couldn't distinguish circle and ellipse by cubic model.
- ② By cubic model, children in any years could scarcely draw the red upper base as ellipse. (Table 5) For the transformation of red upper base from circle to ellipse, they must change the standpoint from the real circle to the image of sight (ellipse). This manipulation is very difficult for them.

Table5: transformation of red upper base



### 1.4 How children drew the contrary poles?

Table6: The relation of yellow and blue domains



We take the problem, how children acquire the cognition of poles, such as near and distant, overlap and far, embrace and separate. We tried to analyze the relation of the yellow ellipse and blue pentagonal mount in case of the plain model (Figure1).

In the relation of yellow and blue, we can divide ①include – be included, ②appear – disappear, ③bound – separate. To understand the relation children should notice these divisions, especially separation. Children need distinguish overlap and apart to draw figures correctly.

Concerning to draw figures apart and outside, children need to consider the overlap relation of two domains. By means of thinking the contrary poles, they reach to the correct figures in succession (figure1). We find the quality of understanding was getting raised gradually by age. It is especially important for children to discover the contrary poles in one thing or in the relation of the two.

### 1.5 Interlude How children change through the play of games?

Children learn and think mechanism of space while they play an active part in games. They consider, suspect, imagine and devise from teacher's influence. Meanwhile teacher should never inculcate the solution.

[Pole asunder game]

In this group game children will think the contrary relation of things in pole asunder as follows.

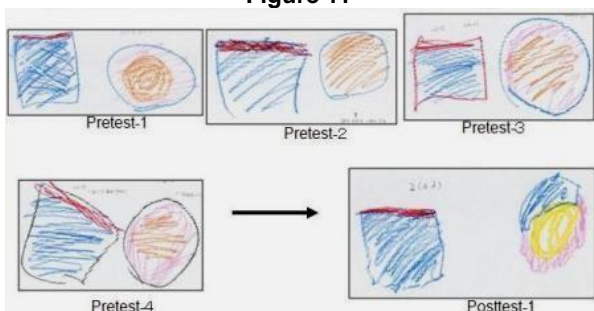
\* In the game children played by the appeal of teacher as follows.

- ① top - bottom
- ② ground – sky charcoal gray - light blue
- ③ candy - cream puff hard - soft
- ④ ant – elephant small - large
- ⑤ long trunk - short leg Dachshund,
- ⑥ left hand – right hand

### 1.6 Change of drawing after the playing

• SIONN (5 years old) drew the model as follows.

Figure 11



SIONN imagined and expressed her asunder poles as follows.

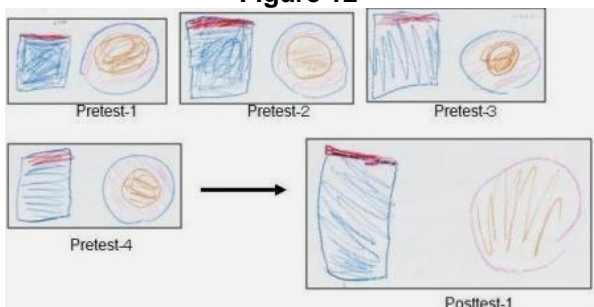
\* sky – sea      solid - liquid  
 airplane – way      unfixd air rout – fixed rout  
 sun – mole      untouched sun – touchable mole

She starts from transforming the red upper base. She drew the contour line of the right model in blue. In the pretest 2, contour line was opened. In the next pretest 3, she drew it with red contour line. In pretest 4, she drew it in black.

In the posttest, she drew it correctly. She changed the understanding of the model from two domains connection to 3 domains connection. It was indeed a splendid change.

• NANAMI (5 years old) drew the model as follow

Figure 12



NANAMI imagined and expressed her asunder poles as follows.

\* sky – sea  
 ground – rainbow  
 ground

Her drawing showed a sort of retrogression in the posttest, ie. she changed to draw the model from three to two domains. What is the reason?

The aspects of attentions to the models differ by each child. The change of recognition through the pretest to the posttest tempts a motivation in children's studying. But this change is not one to one correspondence. The difficulty lies in the narrowness of their understanding. In case NANAMI, passwords are related only to one dimension.

In the studying there are two points. One is child's effort and will, the other is teacher's influence.

## 2 Analysis of drawing of children

### 2.1 Domains

How many domains are there in their drawings?

Table 7: the number of domains of the right cup

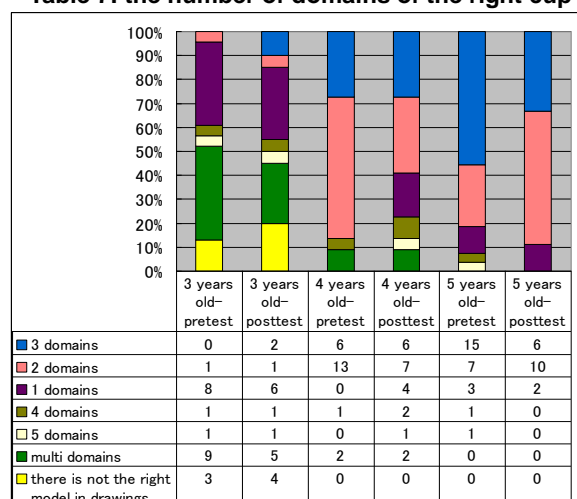


Figure 13

SATOSHI (4 year's old)

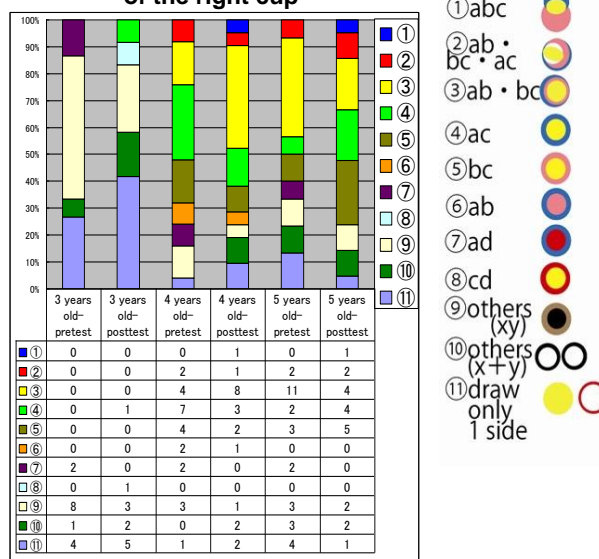


Figure 14

HIKARU (5 year's old)



Table 8: The types of the way to connect domains of the right cup



The cubic model (the right model) has 3 domains. SATOSHI (4 year's old) drew the model as figure13. Though he drew three domains in figure13, it shows different level compared to figure14 (HIKARU 5 year's old).

We will try to classify the types of the way to connect two domains. (Table 7, Right cup model). In this case, there are 3 colors and 3 domains (①-③).

In 3 year's children, there is no relationship in the two domains. In 4 and 5 year's children, there appear some connections in two domains. The quality of understanding is getting gradually by age. Besides the quality of understanding is outstanding in posttest.

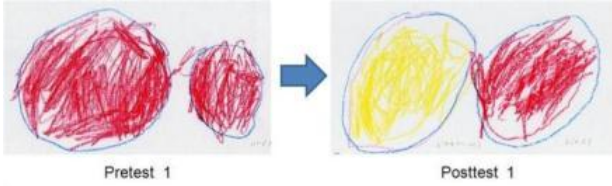
In pretest, some children (4 and 5 children) drew red domain. But they showed changes of



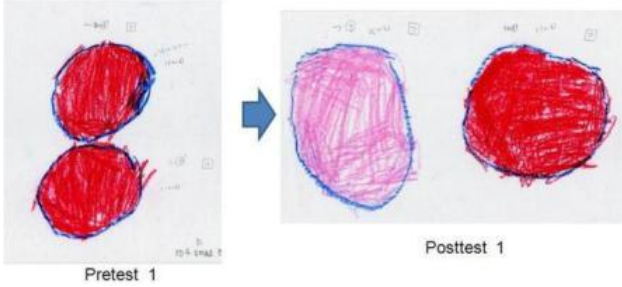
cognition in the posttest as follows.

\* a: blue domain, b: pink domain, c: yellow domain, d: red domain. For example, ac means the combination of blue domain and yellow domain.

**Figure 15: SOYOGU (5 year's) From ad to ab, bc**



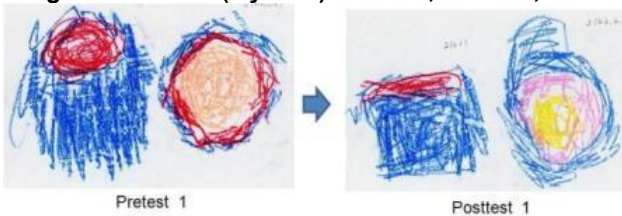
**Figure 16: SEITIROU (4 year's) From ad to ab**



**Figure 17: JYUN (4 year's) From ad to ac**



**Figure 18: ASITA (5 year's) From ad, cd to ab, bc**



They all changed from ad (d is disappeared). Now they drew the model as ab, ac and bc. Though the connection bc is not the real connection, the drawing with bc shows a advanced level.

\*cd, bd: There is no connection of domains both in the original cup and the right cubic model (d can not be seen).

ad: There is connection of domains in the original cup but no connection in the right cubic model (d can not be seen).

ab: There is connection of domains in the original cup and in the right cubic model (surface and reverse).

bc: There is connection of domains in the original cup and connection in the right cubic model.

ac: There is no connection of domains in the original cup but visual

connection in the right model.

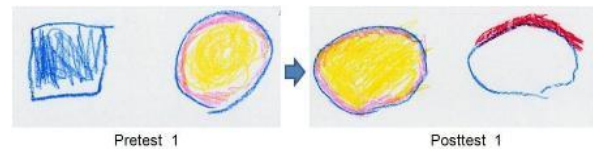
## 2.2 To be or Not to be

Many adults wonder why children draw the red domain, though they cannot see in their eyes. And they will think it is easy to draw the right cubic model connecting yellow and pink domains, while this connection is visible to their eyes. But in fact, children who drew the red domain, have a higher level of cognition. They drew red domain because they understood correctly that the blue domain continues to the reverse red domain. They want to draw what they have understood.

Concerning the left cubic model, a child didn't draw the connection of red and blue domain (Figure19 HONOKA Pretest ). To her eyes the red bottom was certainly visible, but she maybe thought that the bottom should not appear to her eyes.

For children it is rather difficult to draw a model as it appears to their eyes. They draw by their intuition and imagination. So we must by the analysis of their drawings research the ability of sensibility and imagination in children.

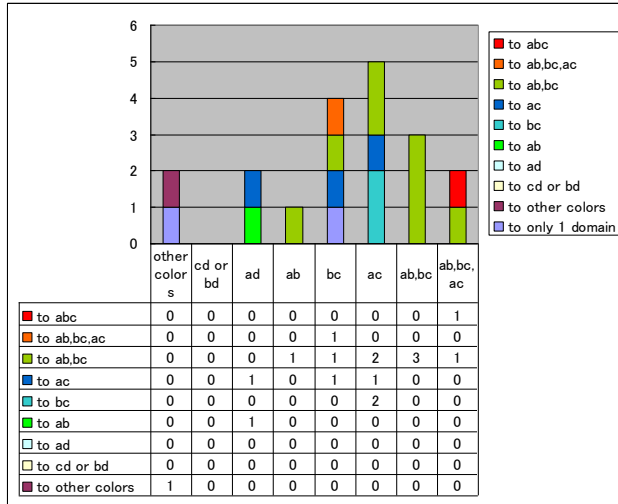
**Figure19: HONOKA (4 year's)**



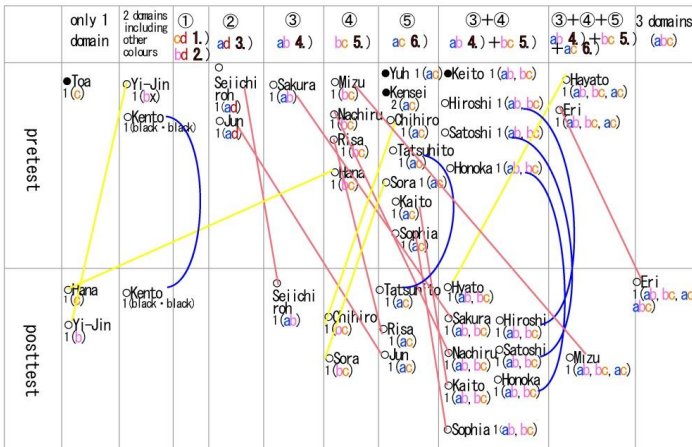
Children draw connections, though they cannot find them under their perspective. And they often correct by themselves through some prompts (in this case, the play of contrary poles). They have since a very early year's many naïve imaginations, and they are apt to see things, not with their eyes, but with their imaginations. Imaginations read cognitions. This ability of imagination by children is very strong and plays decisive effects. They feel things not by sights of things but by images of things. Ability of cognition is characteristic and inherent to children, and as connected with the ability of thinking. They live in the world of imagination, first cognition and then correct it in course of their exercise or praxis. They have ears which can listen the conversation between crow and fox.

Children are fond of drawing what they can understand. Even to their eyes it is 'to be', to their heart it is 'not to be'. And it is 'not to be', it is 'to be'. So, if we pay attention to their process of thinking, we will have a chance to find the secret in the mind of children.

**Table 9: The details of variation of connection of right cup (4 year's)**



**Table 10: The variation of drawings of right cup (4 year's children)**



**The variation of drawings of right cup (5 year's children)**

